TITLE

Two Reliable Methods for Estimating the Mechanical Parameters of a Rotating Three-Inertia System

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Abstract
Two methods for estimating the mechanical parameters of a rotating drive system that can be treated as a resonant 3-inertia system, are proposed. The accuracy of the estimation results is sufficient in order to design a state controller, which damps both resonances actively.

Synopsis
The mechanics of rotating drive systems normally consist of several inertias, which are joined together by elastic shafts. That is why they are resonant and only weakly damped. In order to control such systems, the mechanical parameters have to be known. In literature [ljung], [söderström], [isermann] several estimation methods are well known. The lowest resonant frequencies of known systems [wolff], [singer], [lorenz] are less than 100 Hz. Normally, those systems have been treated as 2-inertia systems. Especially in [singer], moreover one inertia has been known before and the positions both of the motor and the load have been evaluated. The resonant frequencies of modern high-dynamic drive systems are often much higher than 100 Hz. The dynamic range of modern control methods, e.g. DMTC (Direct Mean Torque Control) [flach], makes it possible to damp resonances of several 100 Hz, actively [pahlke]. Therefore, the mechanical parameters such systems have to be known accurately enough.

Experimental set-ups

The two proposed estimation methods have been tested with two experimental set-ups. Each set-up consists of a motor, which is joined to a load by a coupling and a shaft. An incremental encoder is mounted at that side of the motor, which is not joined to the load. The motor of the first set-up [müller], [pahlke] is an induction motor, which is controlled by a DMTC [flach]. The load is a separately excited d.c. machine. The lowest resonant frequencies of the set-up are approximately 394 Hz, 855 Hz, 1975 Hz, . . . . Both the motor and the load of the second set-up are permanently excited synchronous machines. The lowest resonant frequencies of that set-up are approximately 1000 Hz, 1370 Hz, 2300 Hz, . . . .

What has to be payed attention for in particular?
The order of a real system is normally higher than the order of the model, which is assumed for the estimation and for controlling. In the estimation, all time constants, which are not modelled, appear as systematic or correlated errors. As the Least-Squares-estimator reacts very sensitive to systematic errors, they should be avoided by choosing an order of the model, which contains all time constants of the real system. For the identification of a set-up like the investigated ones in this contribution, it is not sufficient to assume a model of order 4. It is necessary to use a model of order 6 as a minimum, better 8, because of the resonance due to the incremental
encoder, which is a fourth inertia, and possible other parasitic time constants. Additionally it is
normally not sufficient to use a simple ARX-model (Auto-Regression with eXtra inputs) without
any error-filter, because the generalised error which is minimized is normally correlated, even if
the measured signals only contain white (not correlated) noise. For instance assume the model
shown in figure 2. Assume the output signal $y(k) = y_0(k) + n(k)$ consisting of a noise-free part
$y_0(k)$ and a white noise $n(k)$. The generalised error $e(k)$ is calculated by

$$e(k) = \sum_{i=0}^{m} \left( \hat{a}_i (y_0(k-i) + n(k-i)) - \hat{b}_i u(k-i) \right)$$

The auto-correlation function of the generalised error is

$$\phi_{ee}^N(\tau) = \frac{1}{N} \sum_{k=0}^{N-1} e(k) \cdot e(k+\tau)$$

$$= \ldots = \frac{1}{N} \sum_{i=0}^{m} \sum_{k=0}^{N-1-i} \hat{a}_i \hat{a}_{i+\tau} \cdot n^2(k-i)$$

$\neq 0$ for $\tau \neq 0$

Using an ARMAX-model (Auto-Regression with Moving Average and eXtra inputs) reduces problems due to this effect.

**Two reliable methods**

In this contribution, two methods for the estimation of the mechanical parameters of a 3-inertia
system are proposed: the classical identification method (ARMAX-model) and a method, which
evaluates the FFT (Fast Fourier Transformation) of the system response by graphical aspects.
Both methods make use of the same measured data. The system was excited by a superposition
of several sinusoidal signals at different frequencies, different phases and equal amplitudes. The
interesting frequency range was chosen to 50 . . . 2000 Hz. The spacing between two successive
frequencies was chosen to $\frac{1}{3}$ Hz. The signals of a pass of 30 seconds were sampled, stored and eval-
uated. Both proposed methods yield the dominant poles and zeros of the transfer function. From
the poles, zeros and the total inertia, which is either known from the construction or measured
during an acceleration test, the mechanical parameters are calculated. If damping and nonlinear-
earities like friction and looseness are neglected, a single-valued conjunction between the transfer
function and the mechanical parameters exists. This conjunction can be solved analytically to the
mechanical parameters, so that iterative methods are not necessary. The calculation only consists
of a few steps. Difficulties concerning convergence, as they can occur with iterative methods, if
unsuitable start values are chosen, do not exist. Both estimation methods use the angular accelera-
tion, which is calculated from the angular position signal by calculating the differential equation
twice.

**The classical method: LS-estimation with ARMAX-model**

The first proposed method is the LS-estimation ([isermann], [ljung], [söderström]), which estimates
the discrete transfer-function by minimizing the sum of the squares of the prediction error. At
the experimental set-ups, the best results have been achieved, if the order of the model has been
assumed to 8 or 10. The same order has been chosen for the error model. A dead time of two
samples has been chosen. The poles and the zeros are extracted from the estimated transfer
function. From them, those poles and zeros, which are close enough to the imaginary axis, are
regarded as dominant.
The graphical method: min-max-search in the FFT

The second proposed method evaluates the FFT of the angular acceleration signal by graphical aspects. Therefore it is supposed that the system to be identified is only weakly damped, and in consequence the FFT shows clear maximum and minimum values. It is required that the spectrum of the exciting torque is constant in the whole relevant frequency range. From the system response, here the angular acceleration, the FFT, then the logarithm is calculated and filtered by a low-pass filter with an edge frequency of $0.01 \cdot f_A$. In order to avoid a shifting of the filtered FFT, the filtering is performed twice – in forward and in backward direction. The filtered FFT nearly only contains the dominant extreme values, while the small ones are smoothed away. Afterwards a simple algorithm searches in steps for all local extreme values. As also extreme values are found, which are not dominant, but have not yet been removed by the filter, they are removed in the following, if they are too much below the absolute maximum or too much above the absolute minimum, or if they do not differ enough from their neighbours in frequency and amplitude.

Experimental results

Both methods have been tested at the previously described set-ups. Figure 3 shows the FFT of the angular acceleration calculated from the angular position of the first set-up. The poles at approx. 397 Hz and approx. 863 Hz, as well as the zeros at approx. 153 Hz and approx. 775 Hz can be seen clearly. Several experiments using excitations of different amplitudes and at standstill as well as with a superposed rotation under load have been performed.

Figure 4 shows the estimated poles (“P”), zeros (“Z”), inertias (“J”) and torsional stiffnesses (“c”) from several experiments. The black bars show the range of the estimation results. The index “A” indicates the results from the first estimation method, the index “F” those from the second one.

Figure 3: FFT of first set-up

Figure 4: Estimation results, first set-up
Figure 5 shows the FFT of the system response of the second set-up. A rotation of $5\frac{1}{2}$° has been superposed to the excitation. It can be noticed that no zero is placed between the two poles. This means that the second set-up cannot be treated as an oscillator motor-coupling-load, but rather as an oscillator sensor-motor-load, because the junctions motor-coupling and coupling-load are much stiffer than the junction sensor-motor. In this case, the mechanical parameters cannot be calculated without previous knowledge. One inertia has to be known. The inertia of the incremental sensor is most reliably known from the construction and was used.

Figure 6 shows the results from several experiments. Without additional rotation the first resonance frequency is approx. 10 Hz higher than with additional rotation. The cause for this behaviour is the notch torque deriving from the load, which is a permanently excited synchronous machine.

Obviously, the parameters of the investigated set-ups are reliably estimated with both methods. With these results it is possible to design a state controller in order to damp the resonances actively ([pahlke]). The advantage of the classical estimation method is, that it can be fitted to other systems, which are e.g. damped very much or which have poles on the real axis. On the other hand, the graphical method takes much less calculation time and is less sensitive to disturbance.
References


[müller] I. Müller, Estimation of the Mechanical Parameters of a stiff Three-Inertia-Drive, PCIM 2001

